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Laboratory Manual

General Physics - part I

Practical Exercises

UNIVERSITY OF CHEMICAL TECHNOLOGY AND METALLURGY

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This manual is intended for the students from the specialization "Metallurgy", taught in English, at the University of Chemical Technology and Metallurgy. However, it may be also useful to students from other specializations, as well from other technical universities. It encompasses a brief survey on the technology of performing the measurement of different physical quantities and the estimation of experimental errors. Here are also included 9 laboratory exercises dedicated to topics from the teaching program in Physics I, i.e. mechanics of solids and fluids, electricity, electromagnetism and optics.

The authors would like to express their gratitude to Assoc. Prof. Dr. Ivailo Gugov for the review and the comments on our work. We appreciate the help and all useful remarks and suggestions during the process of preparation of the manual from all colleagues at the Physics department of UCTM.

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<h1 style="text-align:center">Measurement of Physical Quantities.
Observational Errors.</h1>

I. Physical Quantities.

1. Physical quantity – represents the quantified measure of a physical property of a body or phenomenon.

2. Designation of Physical Quantities:

- **Symbol** – just a matter of convention – one and the same physical quantity may be designated by different symbols, but usually for convenience a common designation is accepted. Actually, symbols for quantities should be chosen according to the international recommendations from ISO 80000, the IUPAP red book and the IUPAC green book. For example, the recommended symbol for the physical quantity 'mass' is m;
- **Numerical value**;
- **Measuring unit (dimension)**.

Example: length L=1m,   L=100cm,   L=1000mm.

II. Ways to measure physical quantities.

1. Direct measurement – the physical quantity value is read directly from the display of the device.

Examples: temperature is measured by thermometer, time by clock, electrical current by ammeter, etc.

2. Indirect measurement – the value of the measured physical quantity is determined by means of a formula or relation among it and other, experimentally measured, physical quantities.

Example: density of a rigid body, $\rho = \dfrac{m}{V}$, where the mass m is measured by scales and the volume, V – determined from the geometry of the body.

III. Measuring units in SI (System International).

1. Base quantities and units:

length – m (Meter)
mass – kg (Kilogram)
time – s (Second)
temperature – K (Kelvin)
electric current – A (Ampere)
amount of substance – mol (Mol)
luminous intensity – cd (Candela)

2. Subsidiary quantities and units:

plane angle – rad (radian)
solid angle– sr (steradian)

3. Derived quantities and units – all other quantities are derived quantities since their dimensions are derived from those of base quantities by multiplication and division. For example, the physical quantity velocity is derived from base quantities length and time and has dimension L/T: $v = \dfrac{s}{t} \Rightarrow$ [m.s^{-1}]. Another example is the unit of force, which has its own name, Newton:

$$F = ma \Rightarrow [\text{kg.m.s}^{-2}] \equiv [N]$$

Some derived physical quantities have dimension 1 and are said to be dimensionless quantities.

4. Prefixes used for the designations of multiples and fractions of the measuring units.

The International System of Units (SI) specifies a set of unit prefixes known as **SI prefixes** or **metric prefixes**. An SI prefix is a name that precedes a basic unit of measure to indicate a decadic multiple or fraction of the unit. Each prefix has a unique symbol that is prepended to the unit symbol. The SI prefixes are standardized by the International Bureau of Weights and Measures in resolutions dating from 1960 to 1991.

Prefix	Symbol	1000^m	10^n	Decimal	Short scale	Long scale
yotta	Y	1000^8	10^{24}	1000000000000000000000000	Septillion	Quadrillion
zetta	Z	1000^7	10^{21}	1000000000000000000000	Sextillion	Trilliard
Exa	E	1000^6	10^{18}	1000000000000000000	Quintillion	Trillion
peta	P	1000^5	10^{15}	1000000000000000	Quadrillion	Billiard
Tera	T	1000^4	10^{12}	1000000000000	Trillion	Billion
giga	G	1000^3	10^9	1000000000	Billion	Milliard
mega	M	1000^2	10^6	1000000	Million	
Kilo	k	1000^1	10^3	1000	Thousand	
hecto	h	1000$^{2/3}$	10^2	100	Hundred	
deca	da	1000$^{1/3}$	10^1	10	Ten	
		1000^0	10^0	1	One	
deci	d	1000$^{-1/3}$	10^{-1}	0.1	Tenth	
centi	c	1000$^{-2/3}$	10^{-2}	0.01	Hundredth	
milli	m	1000^{-1}	10^{-3}	0.001	Thousandth	
micro	μ	1000^{-2}	10^{-6}	0.000001	Millionth	
nano	n	1000^{-3}	10^{-9}	0.000000001	Billionth	Milliardth
pico	p	1000^{-4}	10^{-12}	0.000000000001	Trillionth	Billionth
femto	f	1000^{-5}	10^{-15}	0.000000000000001	Quadrillionth	Billiardth
Atto	a	1000^{-6}	10^{-18}	0.000000000000000001	Quintillionth	Trillionth
zepto	z	1000^{-7}	10^{-21}	0.000000000000000000001	Sextillionth	Trilliardth
yocto	y	1000^{-8}	10^{-24}	0.000000000000000000000001	Septillionth	Quadrillionth

Example: S=5km=5.10^3m
d=6nm=6.10^{-9}m

5. Dimensionless quantities – obtained as a result of the division of two quantities with the same dimension.

IV. Errors occurring during the measurement of physical quantities.

Experience has shown that no measurement, no matter how carefully made, can be completely free of uncertainties. There is always an uncertainty associated with any measured quantity even in the most carefully done experiment and despite using the most sophisticated instruments. This uncertainty in the measured value is known as the error in that particular measured quantity. In presenting experimental results it is very important to objectively estimate the error in the measured result. Such an exercise is very basic to experimental science.

In science, the word error does not carry the usual connotations of the term mistake. As such, errors are not mistakes; you cannot eliminate them by being very careful. The best we can hope to do is to ensure that errors are as small as reasonably possible and to have a reliable estimate of how large they are.

When a measurement of a physical quantity is repeated, the results of the various measurements will, in general, spread over a range of values. This spread in the measured results is due to the errors in the experiment. **Error** of the measured quantity x, denoted usually by Δx, is the difference between the true and the measured value x:

$$\Delta x = x_{true} - x$$

Accuracy is the closeness of agreement between a measured value and the true value. **Precision** refers to the repeatability of measurement. It does not require us to know the correct or true value and shows how tightly repeated measurements cluster around their average value. **Uncertainty** of a measured value is an interval around that value such that any repetition of the measurement will produce a new result that lies within this interval. **Bias** is the difference between the average value of the large series of measurements and the accepted true. **Blunders** are actual mistakes, such as reading an instrument pointer on the wrong scale.

Thus, the result of any physical measurement has three **essential** components:

- a numerical **value** giving the best estimate possible of the quantity measured,
- the degree of **uncertainty** associated with this estimated value and
- the **unit** of the measured quantity.

V. Type of errors

Errors are generally classified into three types:

- **blunders (coarse** errors),
- **systematic** (or determinate) errors and
- **random** (or indeterminate) errors.

A coarse error (blunder) is observed when the measured value is much larger or smaller than the true value of the physical quantity. Blunders are result from carelessness, lack of knowledge or concentration during the laboratory work and the only way to eliminate them

Figure 1

is to repeat the measurement. Systematic error is an error, which is constant throughout the experiment. Systematic errors lead to clustering of the measured values around a value displaced from the "true" value of the quantity. Blunders and systematic errors are actually mistakes which can be eliminated if the experiment methodology is appropriately chosen and the measuring procedure is carefully done.

Random errors on the other hand, can be positive or negative and lead to a dispersion of the measurements around a mean value. For example, in a time period measurement, errors in starting and stopping the clock will lead to random errors, while a defect of the watch causes a systematic error.

The random uncertainties can be treated statistically in order to obtain an unbiased estimation of the true value, the systematic uncertainties cannot. It is seen in the figure the measured six quantities $x_1, x_2, x_3, x_4, x_5, x_6$ (open circles in the figure) are uniformly grouped from both sides of the true value x (full circle in the figure), and in the case of random errors the mean value $\bar{x}$ is close to the true one, and we can even estimate how close they are. This is not possible in the case of systematic errors as it seen from the figure, where x and $\bar{x}$ are far apart.

The error Δx is called absolute error. It is measured in the same units as the measured quantity x. The ratio $\dfrac{\Delta x}{x}$ is dimensionless and is called relative error. It is small quantity usually, that is why it is presented in percents: $\dfrac{\Delta x}{x} \bullet 100\%$.
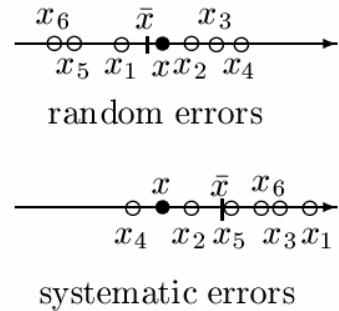
VI. Methods of data processing.

1. Direct measurements.
Any measurement when we measure by a proper device some quantity is called direct measurement.

1.1. Single measurement.
The measuring instruments are never able to help us obtain absolutely exact measurements of any quantity, no matter how carefully made or how sophisticated the instruments may be. Every measuring instrument has a limit to its *precision*. For instance, smallest divisions of the meter stick are millimeters. As an absolute error for any measuring device its smallest division is taken.

1.2. Multiple measurements.
Many experiments involve a series of repeated measurements of a quantity. Let we have measured the quantity *x* and suppose the errors are random only. We collect our data in a table:

x_i	Δx_i	Δx_i^2
x_1	$\overline{x} - x_1$	$(\overline{x} - x_1)^2$
x_2	$\overline{x} - x_2$	$(\overline{x} - x_2)^2$
…	...	…
x_n	$\overline{x} - x_n$	$(\overline{x} - x_n)^2$

1.) The best estimate for *x* is usually its average or mean value.

$$\overline{x} = \frac{1}{n} \sum_{i=1}^{n} x_i .$$

2.) The second column in the table is the deviation or residual of each measurement:

$$\Delta x_i = \overline{x} - x_i .$$

If the deviations are all very small, the measured values are all close to each other and presumably very precise. If some of the deviations are large, the measurements are obviously not so precise.

Some of these differences are negative, other are positive. If the errors are randomly distributed the number of the positive must be approximately the same as the number of positive differences.

3.) In the third column of the table we put square of the errors from the second column:

$$\sigma^2 = \Delta x_i^2 = (\overline{x} - x_i)^2 .$$

4.) The standard deviation or standard error of the mean is:

$$\Delta x = \sqrt{\frac{\sum\limits_{i=1}^{n}\left(\overline{x}-x_i\right)^2}{n(n-1)}}$$

For convenience, the uncertainty $\sigma = \Delta x$ is always defined to be positive, so that $\overline{x} + \Delta x$ is always the highest probable value of the measured quantity and $\overline{x} - \Delta x$ the lowest. Unfortunately, in most scientific measurements we can not be absolutely sure where the measured quantity lies. Choosing this value for $\sigma = \Delta x$ we are sure that the actual quantity lies within the range $\overline{x} \pm \Delta x$ with an approximately 68% probability. Usually, for the visualization of the measured data in case of predominantly random errors occurring, the graph of the distribution of the estimated values of the quantity x is given.
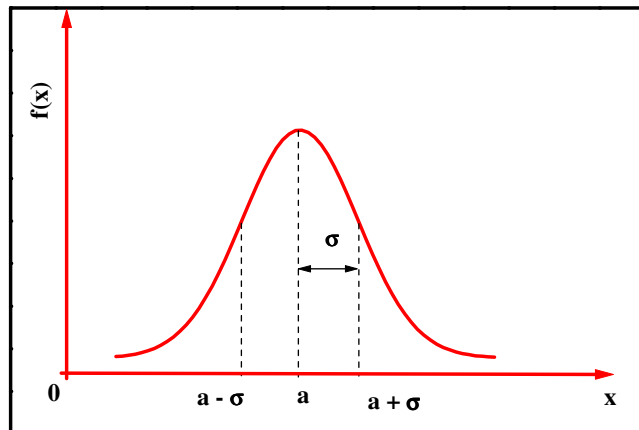
Figure 2. Gauss distribution function

If the measurement results are uniformly distributed around the mean value $a = \overline{x} = \frac{1}{n}\sum\limits_{i=1}^{n} x_i$, then the distribution is named normal or Gauss distribution. F(x) is termed the distribution function and in case of Gauss distribution represents a bell curve which is mathematically described by the formula:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\overline{x})^2}{2\sigma^2}}$$

f(x) – distribution function;

σ - standard deviation, which is the half-width of the bell curve at the inflexion point;

$\bar{x}$ - mean value of the measured quantity x.

It is seen that increasing of the number of the measurements will decrease the error of the mean. In such a manner we can decrease the random error, but we can not influence the device error. If however only a few measurements are performed then the statistically estimated standard deviation is not representative for the accuracy of the end result. Here the normal distribution of the measurement results for the values of the quantity x should be substituted by another statistical distribution – the Student distribution, as shown in the following figure:
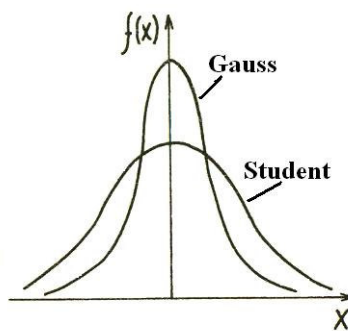

Figure 3. Gauss (normal) and Student distributions

In such cases of smaller number of measurements, a correction of the calculated standard deviation σ is needed by means of the Student coefficient, s_t which is a coefficient dependent on n, the number of measurements done and the probability α, as seen in the table below. α is equal to the probability the true value of x to belong to the interval $(\bar{x} - k\sigma, \bar{x} + k\sigma)$, where k = 1, 2, …

n	α	0.5	0.6	0.7	0.8	0.9	0.95	0.99	0.999
2		1.00	1.38	2.00	3.10	6.30	12.70	63.70	636.30
3		0.82	1.06	1.30	1.90	2.90	4.30	9.90	31.60
4		0.77	0.98	1.30	1.60	2.40	3.20	5.80	12.90
5		0.74	0.94	1.20	1.50	2.10	2.80	4.60	8.60
6		0.72	0.90	1.10	1.40	1.90	2.40	3.70	6.00
10		0.70	0.88	1.10	1.40	1.80	2.30	3.30	4.80
15		0.69	0.87	1.10	1.30	1.80	2.10	3.00	4.10
20		0.69	0.86	1.10	1,30	1.70	2.10	2.90	3.90
∞		0.67	0.84	1.00	1.30	1.60	2.00	2.60	3.30

Thus, the observational error becomes:

$$\Delta x_{corrected} = s_t \sigma$$

2. Indirect measurements.

Frequently, the result of an experiment can not be measured directly. They can be calculated from several measured physical quantities (each of which has a mean value and an error). What is the resulting error in the final result of such an experiment?

If various quantities $x_1, x_2 \ldots x_n$ are measured with small uncertainties $\Delta x_1, \Delta x_2 \ldots \Delta x_n$ and the obtained values are used to calculate some quantity y, which depends on them - $y(x_1, x_2 \ldots x_n)$ - then the uncertainties in x_i cause an uncertainty in y. We can expand $y(x_1 + \Delta x_1, x_2 + \Delta x_2 \ldots x_n + \Delta x_n)$ in series:

$$y(x_1 + \Delta x_1, x_2 + \Delta x_2 \ldots x_n + \Delta x_n) = y(x_1, x_2 \ldots x_n) + \sum_{i=1}^{n} \frac{\partial y}{\partial x_i} \Delta x_i$$

and neglect the terms of second and higher order. The error of y in this approximation is:

$$\Delta y = \sum_{i=1}^{n} \left| \frac{\partial y}{\partial x_i} \right| \Delta x_i$$

We get here the absolute value, because some derivatives can be negative, the uncertainties Δx_i are positive (by definition, but the errors are not) and the maximum error Δy is achieved in this way. The formula above is termed 'propagation of errors'.

If you forget the above formula, a simple method exists, allowing to find the relative error $\frac{\Delta y}{y}$ by following the next steps:

1. Calculate the natural logarithm from $y(x_1, x_2 \ldots x_n)$.
2. Take the full differential of the expression obtained.
3. Take an absolute value of each derivative.
4. Replace the differentials $dx_1, dx_2 \ldots dx_n$ by the uncertainties $\Delta x_1, \Delta x_2 \ldots \Delta x_n$

The formula $y(x_1, x_2 \ldots x_n)$ usually is suitable to take a logarithm, because it has a structure of products of powers of the variables. If some relative error is small (one or two orders smaller than the other errors) it can be neglected in the final result.

3. Rules to present the final result.

The number of reliably known digits in a number is called the number of significant figures. The quantity Δx is an estimate of an uncertainty and

obviously it should not be stated with too much precision, the same is true for the mean value.

1.) If the result is very small or very big number, it is presented in scientific notation:

$$\bar{x} = 263542.17 = 2.6354217 \cdot 10^5.$$

2.) It is enough to keep only three digits after the decimal point:

$$\bar{x} = 2.635 \cdot 10^5.$$

3.) The absolute error must be presented in the same notation as measured quantity:

$$\Delta x = 4256.17 = 0.0425617 \cdot 10^5.$$

4.) The absolute error must be rounded to the first significant figure:

$$\Delta x = 4256.17 = 0.04 \cdot 10^5.$$

5.) In the final result all significant figures except the last one are correct:

$$\bar{x} = 2.64 \cdot 10^5.$$

6.) The final result must be written in the form:

$$\bar{x} = (2.64 \pm 0.04) \cdot 10^5.$$

4. Rules for reporting uncertainties.

1. The result is presented in the form $(x \pm \Delta x)$ [units], for example:

   X = (7.35±0.02) cm

   Note the use of parentheses to apply the unit to both parts.

2. Commonly, only the significant figures are reported, without an explicit uncertainty. This implies that the uncertainty is 1 in the last decimal place.

   For example reporting the result x = 7.35 cm implies uncertainty 0.01 cm. Note that writing 7.352786 cm, when the uncertainty is actually 0.01 cm, is wrong.

3. A special case arises when we have a situation like 1500±100. Scientific notation allows use of a simplified form, reporting the result as 1.5×10^3. In the case of a much smaller uncertainty, 1500±1, we report the result as 1.500×10^3, showing that the zeros on the right are meaningful.

VII. Graphical representation of data.

In experimental physics, the graph of the experimental data is most important in improving the understanding of the experimental results. Moreover from the graphs one can calculate unknown quantities related to the experiments and to compare the experimental data with the theoretical curve

when they are presented on same graph. There is a specific sequence of steps to follow in preparing a graph.

Arrange the data to be plotted in a table.
1. Decide which quantity is to be plotted on the x-axis (the abscissa), usually the independent variable, and which on the y-axis (the ordinate), usually the dependent variable.
2. Use a Cartesian coordinate system.
3. Decide whether or not the origin is to appear on the graph. Some uses of graphs require the origin to appear, even though it is not actually part of the data, for example, if an intercept is to be determined.
4. Choose a scale for each axis, that is, how many units on each axis represent a convenient number of the units of the variable represented on that axis. (Example: 5 divisions = 25 cm) Scales should be chosen so that the data span almost all of the graph paper, and also make it easy to locate arbitrary quantities on the graph. (Example: 5 divisions = 23 cm is a poor choice.) Label the major divisions on each axis.
5. Write a label in the margin next to each axis which indicates the quantity being represented and its units. Write a label in the margin at the top of the graph that indicates the nature of the graph, and the date the data were collected. (Example: "Resistance of a copper wire")
6. Plot each point. The recommended style is a full circle. A cross or triangle may also be used.
7. Draw a smooth curve that comes reasonably close to all of the points. Whenever possible we plot the data or simple functions of the data so that a straight line is expected. Do not simply connect the dots.
8. If the slope of the line is to be determined, choose two points on the line whose values are easily read and that span almost the full width of the graph. These points should not be original data points. Remember that the slope has units that are the ratio of the units on the two axes.

VIII. Least Squares Fitting (Linear Regression).

One of the most common and interesting types of experiment involves the measurement of several values of two (or more) different physical variables to investigate the mathematical relationship between them. In many cases the experiments of this type are those for which the expected relation is linear. We

consider the two physical variables x and y are connected by a linear relation of the form:

$$y = a + bx$$

The graph of y against x should be a straight line that has slope b and intersects the y axis at $y = a$. If we measure n different values $x_1, x_2...x_n$ and the corresponding values $y_1, y_2...y_n$ and if our measurements were subject to no uncertainties, then each of the points (x_i, y_i) would lie exactly on the line $y = a + bx$.

We suppose that, the measurements of y have uncertainties $\Delta y_1, \Delta y_2...\Delta y_n$ and the uncertainty in our measurements of x is negligible. This assumption is often reasonable, because the uncertainties in one variable often are much larger than that for the other. In **Figure 4** they are shown as error bars at both sides of y_i. We should draw a line through them, as close to the points, as possible. The correspondent procedure is called **fitting**.

If the parameters a and b are chosen to minimize the sum:

$$S = \sum_{i=1}^{n} \left(\frac{y_i - a - bx_i}{\Delta y_i} \right)^2$$

The fit is called least squares method. The division to Δy_i makes the sum dimensionless and serve as a proper weight - the measurement with a high error contributes less then those with a small error.

If the parameters a and b are chosen to minimize the sum :

$$S = \sum_{i=1}^{n} \left(\frac{y_i - a - bx_i}{\Delta y_i} \right)^2$$

The fit is called least squares method.

The sum is a function of two parameters – it's minimum is defined by :

$$\frac{\partial S}{\partial a} = -\sum_{i=1}^{n} \left(\frac{y_i - a - bx_i}{\Delta y_i^2} \right) = 0,$$

$$\frac{\partial S}{\partial b} = -\sum_{i=1}^{n} x_i \left(\frac{y_i - a - bx_i}{\Delta y_i^2} \right) = 0$$
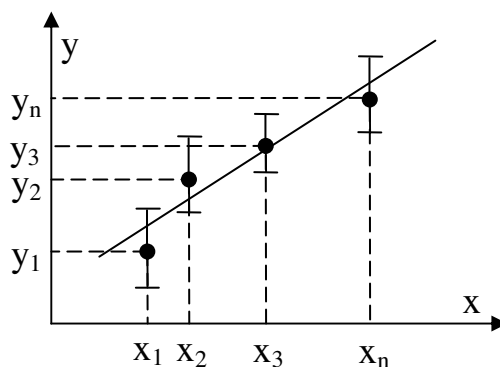

Figure 4. Two measured quantities related by a linear dependence.

The solution of the two linear equation with two unknowns

$$S_1 a + S_x b = S_y, \quad S_x a + S_2 b = S_{xy}$$

is

$$a = \frac{S_y S_2 - S_x S_{xy}}{S_1 S_2 - S_x^2}, \qquad b = \frac{S_1 S_{xy} - S_x S_y}{S_1 S_2 - S_x^2},$$

where S_1, S_x, S_y, S_2 и S_{xy} are :

$$S_1 = \sum_{i=1}^{n}\left(\frac{1}{\Delta y_i^2}\right), \ S_x = \sum_{i=1}^{n}\left(\frac{x_i}{\Delta y_i^2}\right), \ S_y = \sum_{i=1}^{n}\left(\frac{y_i}{\Delta y_i^2}\right), \ S_2 = \sum_{i=1}^{n}\left(\frac{x_i^2}{\Delta y_i^2}\right), \ S_{xy} = \sum_{i=1}^{n}\left(\frac{x_i y_i}{\Delta y_i^2}\right)$$

and the errors of a and b are:

$$\Delta a = \sqrt{\frac{S_2}{S_1 S_2 - S_x^2}}, \ \Delta b = \sqrt{\frac{S_1}{S_1 S_2 - S_x^2}}$$

In the case when all errors of y are equal i.e. $\Delta y_i = \Delta y$, the sums are simplified:

$$S_1 = n, \ \ S_x = \sum_{i=1}^{n} x_i, \ \ S_y = \sum_{i=1}^{n} y_i, \ \ S_2 = \sum_{i=1}^{n} x_i^2, \ \ S_{xy} = \sum_{i=1}^{n} x_i y_i,$$

and the errors are :

$$\Delta a = \frac{\Delta y}{\sqrt{n}}\sqrt{\frac{S_2}{S_2 - S_x^2/n}}, \ \ \Delta b = \frac{\Delta y}{\sqrt{S_2 - S_x^2/n}}$$

It is seen the errors can be decreased, reasonably increasing the number of measurements.
The least squares procedure is implemented nowadays in each graphical program and can be performed just by calling the corresponding procedure after the experimental points are plotted on the graph.

ELASTIC DEFORMATIONS. YOUNG'S MODULUS
OF A STEEL WIRE

THEORETICAL SECTION
The internal molecular forces that tend to resist changes in shape or volume of the condensed matter are collectively called *stress*. Stress can be expressed as the ratio of the applied external force to the area over which the force acts:

$$\text{Stress} = \frac{\text{Force}}{\text{Area}}$$

The simplest type of deformation is the linear extension or shortening. In that case, Hooke's law is fulfilled:

$$\frac{F}{S} = E\frac{\delta L}{L},$$

the quantity **E** is called Young modulus (modulus of linear deformation).

Within the elastic limit, Hooke's law tells us that a force distorts a body by some amount proportional to the force doing the stretching. Recalling the molecular structure of a solid, one can imagine that greater and greater forces are trying to pull the molecules of the solid farther and farther apart (the stretch), which cause greater and greater forces between the molecules trying to pull everything back together again. When a weight on a wire is in equilibrium the downward force exerted by a weight is matched by an upward force exerted by internal forces among the molecules of the string. The Hooke's law is a result of the stable equilibrium of the molecules of the solid. The interaction force between two particles, if the distance between them is changed by **x**, is:

$$F = -\frac{\partial W_p}{\partial x} = -kx,$$

i.e. the force is proportional to the displacement **x**, and the sign "–" designates that the force is always directed to the equilibrium position.

The stress always results in some amount of compression, pulling or twisting of matter. These changes are collectively called *strain*. Strain is the deformation caused by stress. For solids under tension or compression stress, the resulting strain can be expressed as a ration of the resulting change of length to the original, unstressed length:

$$Strain = \frac{\Delta L}{L}$$

Figure 5

The strain in a solid under the influence of external forces leads to change of the shape and dimensions of the solid, i.e to deformation of the solid. Different types of deformations of solids exist. These may be classified as follows:

1. elastic deformations – if the deformation disappear after the external force is removed. The largest stress for which this occurs is called elastic limit.

2. inelastic deformations – the deformation or part of it is retained after removing the external force and lead to irreversible changes in the crystalline
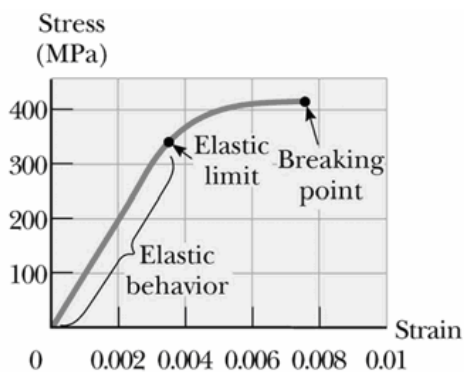
lattice of the solid. When the strain does not return to zero after the stress is removed, the material is said to behave plastically.

EXPERIMENTAL SECTION

I. Estimation of Young's modulus of a steel wire using a dumpy level.

1. The wire is stretched by applying one of the weights. The metal plate of the level tube is brought into horizontal position by a micrometric screw.

2. One of the available weights of mass m is put on the holder and its mass value is written down in the table. The air bubble of the level goes up due to the wire elongation. The micrometric screw is used to restore the horizontal position of the level when the air bubble is in the middle of the level window. The number n of the ticks, needed to turn the screw is also written down in the table.

№	m, kg	G=mg, N	δL=an, m	$k = \dfrac{G}{\delta L}, \dfrac{N}{m}$	$\Delta k, \dfrac{N}{m}$	$\Delta k^2, \left(\dfrac{N}{m}\right)^2$
1.						
2.						
3.						
4.						
5.						
6.						
7.						
8.						

$$\overline{k} = \qquad\qquad \sum_{i=1}^{8}\Delta k_i^2 =$$

Taking into account, that the deforming force in case of a mass m is the weight $G=mg$ (g is the gravitational acceleration), the Young's modulus can be estimated from:

$$E = \frac{GL}{S\delta L}.$$

3. The extension of the wire for a given weight is calculated, having in mind that the force of gravity for a weight with mass m is $G=mg$, according to $\delta L=an$, where a is the micrometer constant (written on the wall next to the experimental setup).

4. All weights are applied one by one to the holder and for each weight the elongation is calculated.

5. The weights are taken away one by one from the holder and the corresponding decrease of the mass is written into the table. The shortening of

the wire is determined in the same way as its elongation but rotating the micrometer screw in the reverse direction. All data is written in the table.

6. The elastic constant of the wire is determined by $k = \dfrac{G}{\delta L}$ and also written down in the table.

7. The average elastic constant $\overline{k}$ of the wire is calculated and the standard deviation $\Delta\overline{k}$ is estimated.

$$\Delta\overline{k} = \pm\sqrt{\dfrac{\sum\limits_{i=1}^{n}\Delta k_i^2}{n(n-1)}} \ , \ n=8$$

$\Delta\overline{k} =$

8. The end result is given by:

$k = \overline{k} \pm \Delta\overline{k}$

9. Young's modulus is calculated by the formula:

$$E = \dfrac{L}{S}\overline{k} = \dfrac{4\,L}{\pi\,d^2}\overline{k} =$$

$E =$

where **L** – initial wire length, and **ΔL** – absolute error with which L is measured (also to be found on the wall next to the experimental set-up);

$L =$

S – surface of the wire cross-section ($S = \dfrac{\pi\,d^2}{4}$) ;

d – diameter of the wire, **Δd** – absolute error of the diameter (to be seen on the list at the wall around).

$d =$

$S =$

10. The relative error with which is estimated the Young's modulus cab be determined from:

$$\varepsilon = \dfrac{\Delta E}{E} = \pm\left(\dfrac{\Delta L}{L} + \dfrac{2\,\Delta d}{d} + \dfrac{\Delta\overline{k}}{\overline{k}}\right),$$

$$\varepsilon = \frac{\Delta E}{E} =$$

11. The absolute error for the Young's modulus is given by:
$$\Delta E = \varepsilon . E$$

$$\Delta E =$$

12. The end result is:
$$E = E \pm \Delta E$$

II. Schematic view of the experimental set-up.
A – steel wire
B – steel holder
G – weights
L – metal plane with the dumpy level
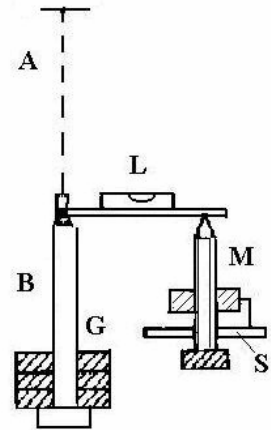M – micrometer screw
S – scale

Figure 6. Experimental set-up

MEASUREMENT OF THE FLUID VISCOSITY USING STOKES' LAW

THEORETICAL SECTION

Fluid could be determined as a substance that deforms continuously when subjected to a shear stress. The property *viscosity* of a fluid relates the shear stress in a fluid to the angular rate of deformation. *Fluid Mechanics* studies the fluid properties.

There are two main types of fluid flow. If the flow is smooth, such that neighbouring layers of the fluid slide by each other smoothly, the flow is said to be streamline or laminar flow. In streamline flow, each particle of the fluid follows a smooth path, called a streamline, and these paths do not cross one another. Above a certain speed, the flow becomes turbulent. Turbulent flow is characterized by small, whirlpool-like circles called eddy currents or eddies.

The relative motion of the layers in a real fluid is accompanied by the certain amount of internal friction, called viscosity. The frictional force in the case of laminar flow is given by the Newton's law:
$$F = \eta \, S \, \frac{\Delta v}{\Delta z},$$

where **η** is the internal friction factor for the fluid (dynamic viscosity) and depends on the type of the fluid and on temperature; **S** – area of the layer, at which the force F is applied; $\dfrac{\Delta v}{\Delta z}$ – gradient of velocity (velocity change **Δv** at a distance **Δz** perpendicular to the motion direction of the fluid layers);

On a spherical body falling free in a fluid, three forces are exerted simultaneously:

1. Force of gravity (weight):
G=mg=ρVg,   where
m – mass of the body ($m = \rho V$);
ρ – density of the body;

V – body volume ($V = \dfrac{4}{3}\pi r^3$);

r – radius of the sphere;
g – the gravitational constant.

2. Buoyancy, which equals the weight of the fluid dispaced by the falling body (Archimedes law):
G₁=m₁g=ρ₁Vg,   where

Figure 7

m₁ –mass of the displaced by the falling body fluid ($m_1 = \rho_1 V$);

ρ₁ – fluid density;
V – volume of the displaced by the body liquid, which equals the volume of the body.

3. Viscosity force, given by the Stokes' law for the friction force exerted on a spherical body:
F=6πηrv , where
F - is the drag force of the fluid on a sphere,
η is the fluid viscosity,
v is the velocity of the sphere relative to the fluid, and
r is the radius of the sphere.

Stokes' Law is fulfilled only under some conditions and thus, a subject to some limitations. Specifically, this relationship is valid only for laminar flow.

In the beginning of its motion, the sphere is accelerating and its velocity increases. While the velocity increases, the viscosity force also increases until the weight **G** is balanced by the sum of the buoyancy **G₁** and the viscosity force **F**. Further, the sphere is falling with a uniform speed in the liquid:

$\vec{G} + \vec{G}_1 + \vec{F} = 0$ and in scalar form: –G+G₁+F=0.

Then the dynamic viscosity of the liquid is obtained from:
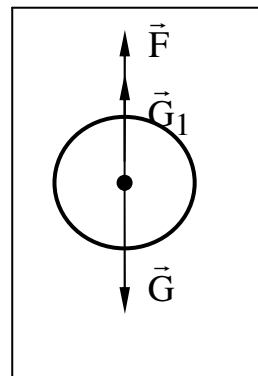
$$\eta = \frac{2}{9}\frac{(\rho - \rho_1)g}{v}r^2.$$

The velocity of the sphere is given by the following relation:

$$v = \frac{L}{t}$$

with L – distance at which the uniformly propagating sphere moves for a time t. The diameter of the sphere is $d = 2r$ and finally we obtain:

$$\eta = \frac{(\rho - \rho_1)g}{18\,L} d^2 t.$$

The last expression is strictly fulfilled only for a body propagating in an infinite fluid, i.e. when the volume of the fluid is much larger than the particle volume. Since the liquid in the vicinity of the cylinder walls is motionless, and the fluid layer at the interface of the liquid and the sphere propagates with the same velocity as the sphere itself, the velocity gradient increases. As a result the velocity v of the sphere in the cylinder becomes smaller compared to that of a sphere propagating in an infinite liquid.

Taking into account the influence of the cylinder walls on the velocity of a sphere propagating in a finite liquid, the dynamic viscosity of the liquid is given by:

$$\eta = \frac{2}{9} \frac{(\rho - \rho_1)g}{v\left(1 + 2,4\dfrac{r}{R}\right)} r^2,$$

where **R** – radius of the cylinder.

Since the diameters of the used spheres are much smaller than the radius of the cylinder, the ratio $2,4\dfrac{r}{R} \approx 0,01$ and it may be neglected in the above relation (only assuming that the sphere propagates along the cylinder axis).

EXPERIMENTAL SECTION

The viscosity of a known liquid will be measured using a cylinder full of fluid (glycerine) as a viscometer. If a small ball is dropped into this liquid it moves with a constant velocity which can be measured by a stopwatch.

The ball is placed in the fluid and the time that it takes to fall the length of the cylinder is recorded. This time is then utilized to back the viscosity out of the velocity relationship that we derived using Stokes' law and summing forces. As the ball is dropped into the fluid it accelerates as a result of the gravitational field until the
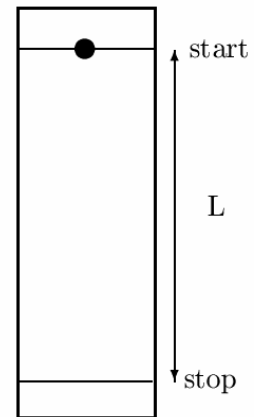

Figure 8. Stokes viscometer

ball reaches constant velocity. Terminal velocity occurs when the viscous and buoyancy forces equal the weight of the ball. At this point the velocity of the ball is maximum, or terminal.

In order to determine the viscosity of a liquid using the Stokes' method, it is required to do the following.

1. Lead spheres with diameter d are used. The diameter of each sphere is measured by a micrometer. (Be careful – lead is a very soft material and deforms easily!).

2. After the sphere diameter is measured, the body is let to fall into the glycerine. The motion should be in the direction of the cylinder axis. The time **t**, required for the sphere to cover the distance **L** is measured by a stopwatch.

3. Ten measurements for different lead spheres are performed and the results are written in the table below.

№	d, mm	d, m	t, s	d^2t, m^{2}s	η, Pa.s	$\Delta\eta$, Pa.s	$\Delta\eta^2$, (Pa.s)^2
1.							
2.							
3.							
4.							
5.							
6.							
7.							
8.							
9.							
10.							

$$\overline{\eta} = \qquad\qquad \sum_{i=1}^{10}\Delta\eta_i^2 =$$

4. The glycerine density ρ_1, the density of lead spheres ρ and the distance **L** are given:

$$\rho = \qquad\qquad \rho_1 = \qquad\qquad L =$$

5. The dynamic viscosity of glycerine is calculated from:

$$\eta = \frac{(\rho - \rho_1)g}{18\,L}\,d^2t\,, \text{ where } \frac{(\rho - \rho_1)g}{18\,L} = k(\text{const}), \text{ i. e. } \eta = k\,d^2t\,.$$

The constant $k =$

6. From the obtained values of the dynamic viscosity, the average viscosity $\overline{\eta}$ and its error $\Delta\overline{\eta}$ is calculated.

$$\Delta\overline{\eta} = \pm\sqrt{\frac{\sum\limits_{i=1}^{n}\Delta\eta_i^2}{n(n-1)}}, \qquad n=10$$

$$\Delta\overline{\eta} =$$

The correction with the Student coefficient, s_t for $\alpha = 0.95$ and $n = 10$ has to be done and the final result should be written.

7. Final result: $\eta = \overline{\eta} \pm (s_s\Delta\overline{\eta})$

MEASUREMENT OF THE LIGHT WAVELENGTHS OF DIFFERENT COLORS APPARATUS

The light wavelength is measured using diffraction grating, lens and screen as shown in the Figure. The lights of different colors are obtained from monochromatic light emitting diodes mounted in torches.
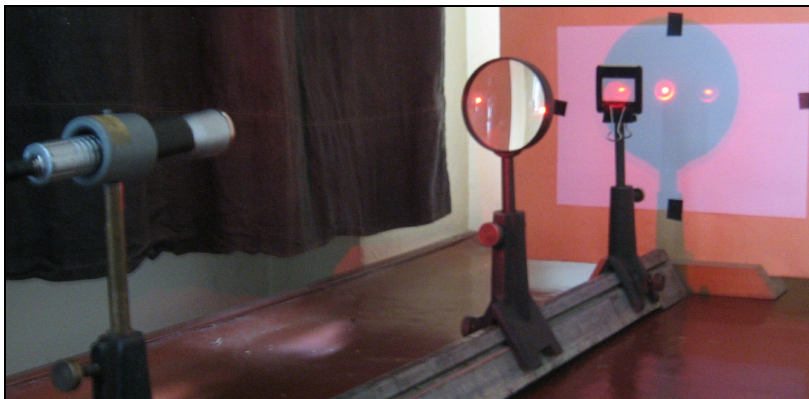
Figure 9. Experimental set-up for measuring the light wavelength.

THEORETICAL SECTION

The wave nature of light manifests in the phenomena of diffraction and interference. They can be explained qualitatively by the Huygens' principle which slates that every point on a wave front can be considered as a source of tiny wavelets that spread out in the forward direction at the speed of the wave itself. The new wave front is the envelope of all the wavelets that is, the tangent to all of them.

A large number of equally spaced parallel slits is called a diffraction grating. Gratings can be made by precision machining of very fine parallel lines on a glass plate. The untouched spaces between the lines serve as slits. Photographic transparencies of an original grating serve as inexpensive gratings. Gratings containing 5-10000 lines per centimeter are common, and are very useful for precise measurements of wavelengths. A diffraction grating containing slits is called a **transmission grating**. Another type of diffraction grating is the **reflection grating**, made by ruling fine lines on a metallic or glass surface from which light is reflected and analyzed.

The simplest way to measure the wavelength is to use the setup shown in Figure 9. Diffractive grating is placed between the lens and the screen and the light from the source is focused on the screen, where the zero and the first maxima are seen.

DERIVATION OF THE FORMULA FOR THE WAVELENTH

The lens L focuses the rays from the source O to the image O`. The optical paths of all rays from the source to it's image are equal, let's denote this path by l. When the grating is between the lens and the screen there will be other images on the screen, due to the diffraction from the grating. The position in point C of the first maximum (the bright spot at distance x from the zero maximum O`) (Figure 10) is a result of the constructive interference – the difference of the optical paths from two neighbouring slits of the grating is equal to the wavelength λ. It is seen from Figure 10a the optical path of the ray through point A is $l - l_1 + l_1'$, and the similar ray through point B is $l - l_2 + l_2'$. Their difference is:

$$(l_2 - l_1) - (l'_2 - l'_1) = \lambda$$
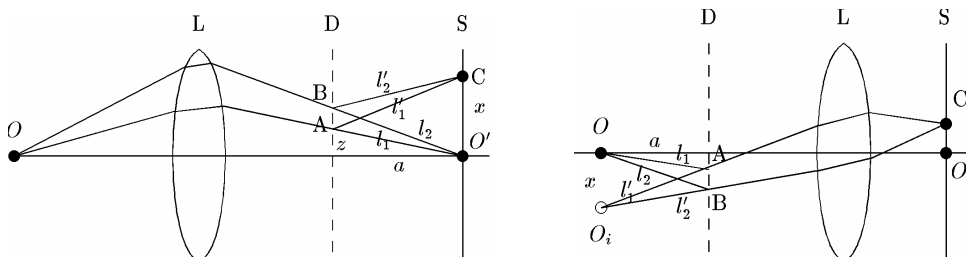
a) The grating between lens and screen.   b) The lens grating between and screen.

Figure 10. Formation of images from light source through lens and diffraction grating.

Here l_1 and l_2 are the lengths of the segments AO` and BO`, and l'_1 and l'_2 are AC and BC. If the distance from point A to the optical axis is z, and the distance of the grating to the screen is a:

$$l_1 = \sqrt{z^2 + a^2} \qquad l_2 = \sqrt{(z+d)^2 + a^2}$$
$$l'_1 = \sqrt{(x-z)^2 + a^2} \qquad l'_2 = \sqrt{(x-z-d)^2 + a^2},$$

d is the constant of the diffraction grating (the segment AB in Figure 10).

The grating constant d is very small (2.10^{-4} cm) compared to x and a (in the order of cm), so we can expand l_2 and l'_2 in Taylor series of d and obtain as a first order approximation by d:

$$l_2 = l_1 + \frac{zd}{l_1} \quad , \qquad l'_2 = l'_1 - \frac{(x-z)d}{l'_1}$$

From the condition of the first maximum:

$$\frac{zd}{l_1} + \frac{(x-z)d}{l'_1} = \lambda.$$

This equation can be solved, taking into account that the ratio $\varepsilon = \lambda/d$ may be considered small:

$$\lambda = \frac{dx}{\sqrt{a^2 + x^2}}$$

If the lens is between the grating and the screen as in Figure 10 the same formula is obtained, but in this case x is the distance of the virtual image O_i from the optical axis, which can not be directly measured.

EXPERIMENTAL SECTION

ASSIGNMENTS

1. Fix the light torch, lens and the diffractive grating in such positions on the optical bench to obtain sharp images on the screen. Use the power supply adaptor to feed the torch, and ruler to measure the distances.
2. Measure the distance a between the grating and the screen and the distance between the two first order maxima. Fill in the table with measured quantities.
3. Change the distance of the grating to the screen and measure again.
4. Change the torch and repeat steps 1 to 3 again.
5. Calculate the wavelength for each measurement.

Color	1) –					2) –				
№	a, m	x, m	λ, m	$\Delta\lambda$,m	$\Delta\lambda^2$, m^2	a, m	x, m	λ, m	$\Delta\lambda$,m	$\Delta\lambda^2$, m^2
1.										
2.										
3.										
4.										
5.										

$$\overline{\lambda}= \qquad \sum_{i=1}^{5}\Delta\lambda_i^2 \qquad \overline{\lambda}= \qquad \sum_{i=1}^{5}\Delta\lambda_i^2$$

6. Calculate the mean value of the wavelength for each color.

$$\overline{\lambda} =\frac{1}{5}\sum_{i=1}^{5}\lambda_i$$

7. Calculate the errors and make the correction with the Student coefficient, s_t for $\alpha = 0.95$:

$$\Delta\lambda =\sqrt{\frac{\sum_{i=1}^{5}\Delta\lambda_i^2}{5.4}}$$

8. Write the final result for each color.

$$\lambda_1 =\overline{\lambda}_1 \pm\Delta\lambda_1 =$$

$$\lambda_2 =\overline{\lambda}_2 \pm\Delta\lambda_2 =$$

MEASURING THE C_P/C_V -RATIO OF AIR
BY CLEMENT-DESORMES METHOD

THEORETICAL SECTION
Molar heat **capacity C** is a thermodynamic quantity equal to the amount of heat to be transferred to 1 mol of a substance in order to increase its temperature with **1К**: $C =\dfrac{dQ}{dT}$.

For gaseous substances the molar heat capacity strongly depends on the temperature and the conditions under which heat is exchanged. For all gases two molar heat capacities are defined: C_P means molar heat capacity at constant pressure (isobaric process) and C_V is the molar heat capacity at constant volume

(isochoric process). The ratio $\kappa = \dfrac{C_p}{C_V}$ is called an adiabatic coefficient and also represents the exponent in the Poisson's formula $pV^{\kappa} = const$ which describes the adiabatic process ($\delta Q = 0$). The value of κ can be obtained utilizing changes in the parameters of a gas contained in a vessel with walls with constant temperature and undergoing particular thermodynamic processes (Clement-Desormes method).

In case of the isochoric process the transfered amount of heat leads only to increase of the gas temperature and no work is done, i.e. only the internal energy of the gas is increased. The First law of thermodynamics is: $\delta Q = dU + dA = dU + p\,dV$ and the formula for the internal energy of 1 mol ideal gas is: $U = \dfrac{i}{2}RT$. The molar heat capacity at a constant volume ($dV=0$) for an ideal gas is given by: $Cv = \dfrac{dQ}{dT} = \dfrac{dU}{dT} = \dfrac{i}{2}R$, where **i** is the number of degrees of freedom of a molecule. Degrees of freedom are the minimum number of independent coordinates which determine the molecule position in space; **R** – universal gas constant.

During the isobaric process the transferred heat both increase the internal energy of the gas and does a work to change the gas volume. Thus, taking into account the First law of thermodynamics and the ideal gas state equation $pV = RT$, the molar heat capacity at constant pressure ($p=const$) is obtained from:

$$Cp = \frac{dQ}{dT} = \frac{dU}{dT} + \frac{p\,dV}{dT} = \frac{i}{2}R + R = Cv + R = \frac{i+2}{2}R\,.$$

For the adiabatic coefficient we have that $\kappa = \dfrac{C_p}{C_V} = \dfrac{i+2}{i}$.

The adiabatic ratio is characteristic for the gas studied and depends only on the number of degrees of freedom of the gas molecules.

Let gas with volume V_1 has pressure $p_1 = p_0 + h_1$ (p_0 – atmospheric pressure) and the temperature is T_1, equal to the room temperature (**I state**). If the gas is quickly introduced to a state, corresponding to atmospheric pressure p_0, it will adiabatically expand to volume V_2 and the temperature will be decreased to $T_2 < T_1$ (**II state**). The vessel with the gas A is big enough, so that the volume loss of gas when adiabatically equalizing the pressures may be

neglected. Thus, we may write: $p_1 V_1^{\kappa} = p_0 V_2^{\kappa}$. Applying the ideal gas state equation for the states I and II, leads to: $\dfrac{p_1 V_1}{T_1} = \dfrac{p_0 V_2}{T_2}$.

Taking into account the last two dependencies, it may be obtained: $\left(\dfrac{p_1}{p_0}\right)^{\kappa-1} = \left(\dfrac{T_1}{T_2}\right)^{\kappa}$. Further, the gas volume is kept constant (V_2=const) and since the gas is heating up, the temperature is increased to the ambient temperature T_1, and the pressure increase from p_0 to $p_2 = p_0 + h_2$ (**III state**). This isochoric transition leads to $\dfrac{p_2}{p_0} = \dfrac{T_1}{T_2}$. Substituting p_1 and p_2 we obtain:

$\left(\dfrac{p_0 + h_1}{p_0}\right)^{\kappa-1} = \left(\dfrac{T_1}{T_2}\right)^{\kappa}$ and thus, $\dfrac{p_0 + h_2}{p_0} = \dfrac{T_1}{T_2}$. After excluding $\dfrac{T_1}{T_2}$ from the above equations, we obtain $\left(\dfrac{p_0 + h_1}{p_0}\right)^{\kappa-1} = \left(\dfrac{p_0 + h_2}{p_0}\right)^{\kappa}$. The last relation may be re-written as follows: $\left(1 + \dfrac{h_1}{p_0}\right)^{\kappa-1} = \left(1 + \dfrac{h_2}{p_0}\right)^{\kappa}$, where the ratios $\dfrac{h_1}{p_0} \ll 1$ and $\dfrac{h_2}{p_0} \ll 1$ are small. If the left and right side of the above equation are expanded in series, and only the terms containing the first power of the ratios $\dfrac{h_1}{p_0}$ and $\dfrac{h_2}{p_0}$ are considered, the final result will be:

$$\kappa = \dfrac{h_1}{h_1 - h_2}$$

From this last relation the adiabatic coefficient κ may be calculated, if the differences in the water heights h_1 and h_2 for the two arms of the U-tube are measured.

EXPERIMENTAL SECTION

In order to measure the adiabatic coefficient follows the next steps:

1. The valve K_1 connecting the air in the vessel with the atmospheric air is opened.

2. Valve K_2, is turned in such a way that it simultaneously connects the pump **P** with the manometer **M** and the vessel **A**. The water level into the two arms of the U-shape manometer **(U-tube)** should be equal in this position of the valve. Thus, the hydrostatic pressure of water into the arms of the manometer is equal to the atmospheric one.

3. Valve K_1 is closed.

4. By pumping the pressure in the vessel A is adiabatically increased compared to the atmospheric one. The pressure increase for the air in vessel A corresponds to a difference between the water heights in the two arms of the U-shape manometer ~20cm. The valve K_2 must be quickly closed – the pump is isolated from the system vessel A and manometer.

5. The pressure in **A** is slowly decreased until the temperature of the air inside the vessel becomes equal to the room temperature. After a few minutes, thermodynamic equilibrium is reached and the difference between the heights of the two arms of the manometer, h_1, can be measured.

6. The valve K_1 is to be quickly rotated at 180^0 (air whistle should be heard). In this way, the air in the vessel A is adiabatically expanded.

7. Meanwhile, the temperature of the air in vessel A has now become lower than the room temperature. The air in A receives some amount of heat from the environment to the air in A until its temperature reaches room temperature. This results in a raise of the pressure in the vessel A and at the same time, a difference h_2, in the water height in the two sleeves of the manometer, which corresponds to an equilibrium state and the end of the process.

8. Having measured the values of h_1 and h_2 the ratio of the molar capacities κ is obtained from the relation. Then the average of the all measured values is calculated.

K_1 – valve
K_2 –valve
A – shielded glass vessel with air
M – manometer
P – pump
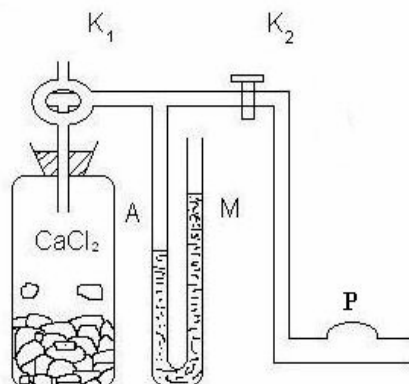
Figure 11. Experimental set-up

№	h_1, mm	h_2, mm	$h_1 - h_2$, mm	κ	$\Delta\kappa$	$\Delta\kappa^2$
1.						
2.						
3.						
4.						
5.						
6.						
7.						
8.						
9.						
10.						
				$\overline{\kappa}=$		$\sum\Delta\kappa_i^2$

9. From the obtained values of κ the average $\overline{\kappa}$ is estimated and the standard deviation $\Delta\kappa$ is calculated.

$$\Delta\kappa = \sqrt{\frac{\sum\limits_{i=1}^{n}\Delta\kappa_i^2}{n(n-1)}}, \quad n=10$$

$\Delta\kappa =$

10. Correction with the Student coefficient for α=0.95 is done for n= 10.

11. End result.
$\kappa = \overline{\kappa} \pm s_t \Delta\kappa$

MEASUREMENT OF THE RESISTANCE AND TEMPERATURE COEFFICIENT OF A METAL BY WHEATSTONE BRIDGE

APPARATUS

The bridge method is widely used in precise measurements, even in the tiny solid state sensors. The Wheatstone bridge is popular bridge circuit used to measure impedances. It is utilized for the measurement of unknown electrical resistances by balancing two legs of a circuit, as shown in Figure 12, called bridge, the one leg of which includes the unknown component, and the other has known variable resistors.

Figure 12.

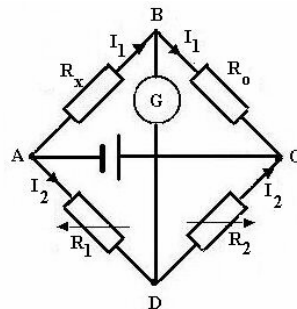In the circuit, R_x is the unknown resistance to be measured; R_1, R_2 and R_0 are resistors of known resistance and the resistance of R_2 is adjustable. In the circuit branch between the points **A** and **C** a supply voltage is applied. R_2 is varied until the current through the galvanometer G becomes zero, then the voltage between the two midpoints (**B** and **D**) will be zero too. It is said that the bridge is balanced when this condition is reached. Detecting zero current can be done to extremely high accuracy, that is why a galvanometer is used. Therefore, if R_1, R_2 and R_0 are known to high precision, then R_x can be measured to high precision. Very small changes in R_x disrupt the balance and are readily detected.

When the bridge is balanced, the same currents I_1 flow trough R_x, R_0 and the current I_2 flows trough R_1, R_2. In this case the voltages on R_x and R_1 are the same and from the Ohm's law $I_1 R_x = I_2 R_1$, and the same is valid for the other two resistance $I_1 R_0 = I_2 R_2$. Dividing two equations we get:

$$R_x = \frac{R_1}{R_2} R_0$$

In the quantum theory of conductivity, the electrical resistance of metals is not due to the collisions of the free electrons with the positively charged ions of the lattice but may be explained as the result of the scattering of the free electrons on the lattice imperfections/defects and on the lattice heat oscillations (the phonons). The result is that in the case of very pure metals the resistance will tend to zero when the temperature approaches the absolute zero (though the latter phenomenon should not be mistaken with superconductivity!).

The dependence of the electrical resistance on temperature is linear, i.e. of the type: $R_t = R_0 (1 + \alpha t)$, where R_t – resistance at temperature t °C, R_0 – resistance of copper at 0 °C, α– thermal coefficient of ohmic resistance measured in $[K^{-1}]$ and is equal to the relative change in the electrical resistance when the temperature is changed by 1 K.

MEASUREMENTS

Measure the temperature dependence of a metal resistor.

1. As a resistor a copper wire is used, dipped into a test tube glass, filled with glycerine or other liquid with low thermal and electrical conductivity (transformer oil). The tube is put on water bath and is heated on by a hot plate. The copper wire resistance is measured by balancing the Wheatstone bridge at different temperatures. The temperature is increased by a step of 5°C starting from room temperature.

2. The measurement results should be put in the table below.

№	t, °C	R, Ω
1.		
2		
3		
4		
5		
6		
7		
8		
9		
10		
	$\bar{t}=$	$\bar{R}=$

3. Draw a graph - on the abscissa put the temperature and on the ordinate put the resistance.

4. Draw a straight line through the experimental points in the graph.

5. Determine the resistance at 0°C by extrapolating the straight line until it intersects the ordinate at t = 0°C and calculate the thermal coefficient of resistance for the copper wire.

The thermal coefficient α is determined by the following relation:

$$\alpha = \frac{\bar{R} - R_0}{R_0 \bar{t}}$$
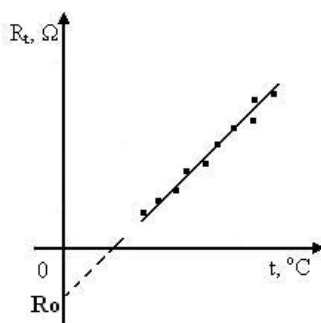
Determine the relative error:

$$\frac{\Delta\alpha}{\alpha} = \frac{\Delta R + \Delta R_0}{\bar{R} - R_0} + \frac{\Delta R_0}{R_0} + \frac{\Delta t}{\bar{t}}$$

6. Calculate the absolute error

$\Delta\alpha =$

Figure 13.

7. The final result is:

$\alpha \pm \Delta\alpha$

MEASUREMENT OF THE ELECTRON SPECIFIC CHARGE USING MAGNETRON TUBE

THEORETICAL SECTION

The specific charge of an electron is the ratio of the electron charge and its mass *e/m*. As magnetron may be regarded a vacuum tube (diode) placed inside a solenoid which can generate a magnetic field $\vec{B}$ coaxially to the tube (see the experimental section). Heating of the diode's cathode causes thermal emission of the electrons from the cathode surface. The released electrons move directly to the anode. When a current flows trough the solenoid, in the space between the cathode and the anode two fields exist: magnetic (from the solenoid, axial to the tube) and electric (from the diode, radial to the anode and the
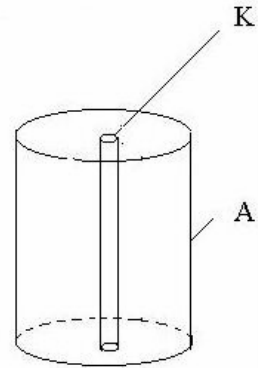
Figure 14. Diode lamp

cathode). We will suppose the magnetic field $\vec{B}$ is homogenous and perpendicular to the electron velocity, $\vec{v}$. In this case the path of the electron is a curve with the curvature radius *R,* reversely proportional to the magnitude of magnetic field $\vec{B}$ (the larger *B*, the smaller the electron path radius), see "Experimental section" and the graph Ia(Is). If the current in the solenoid increases, the magnitude of magnetic field $\vec{B}$ increases too. When it reaches a certain value B_{cr}, some electrons cannot reach the anode and this will result in an abrupt and well recognized decrease of the anode current.

In case of a switched off magnetic field, i.e. $\vec{B} = 0$, the electric field with potential difference U between the heated cathode and the anode transports all electrons to the anode. Electric current, I_a flows to the anode. The work eU of the electric field between the cathode and the anode increase the kinetic energy of the electron, $eU = \dfrac{mv^2}{2}$. Solving this equation for the specific charge, we get $\dfrac{e}{m} = \dfrac{v^2}{2U}$. In order to calculate the specific electron charge **e/m,** it is necessary to measure the applied voltage U and to calculate the velocity v. The voltage is measured by a voltmeter, while the velocity can be determined if the electron motion in the additionally applied homogenous magnetic field of the

coil in the magnetron is regarded. The homogenous magnetic field is acting upon the electron with a Lorentz force, given by:

$$\vec{F} = e\left(\vec{v} \times \vec{B}\right).$$

For small values of the magnetic induction $\vec{B}$ the electron trajectories become non-linear, but the electron is still capable of reaching the anode and an anodic current is flowing, as shown in the graph below. The anode current **Ia** remains almost constant. If the induction $\vec{B}$ increases and reaches some critical value, **B$_{cr}$,** the trajectory curvature of some of the electrons will become large enough and they will not be able to reach the anode, i.e. the anode current will decrease. Taking into account that the Lorentz force is a centripetal force in this case, we can find the velocity of the electron at the critical magnetic field **B$_{cr}$**:

$$\frac{mv^2}{r} = evB_{cr}, \qquad \text{(if } \vec{v} \perp \vec{B} \text{)}.$$

Having in mind that at B$_{cr}$, the curvature radius is **r $\approx$ r$_a$/2** (**r$_a$** – radius of the anode), it may be written for the specific charge: $\dfrac{e}{m} = \dfrac{8U}{r_a^2 B_{cr}^2}$, where the critical magnetic induction of the coil magnetic field is determined by: $B_{cr} = \mu_0 n I_{scr}$, with **I$_{scr}$** being the current which corresponds to the critical value of the magnetic induction, **B$_{cr}$**; **n** – the number of turns per meter length;

μ$_0$ = 4π.10^{-7} H/A^2 – magnetic constant (permeability of vacuum).

The specific charge of the electron can be determined from the following expression:

$$\frac{e}{m} = \frac{8U}{\mu^2 n^2 I_{scr}^2 r_a^2},$$

assuming, it is fulfilled, that the cathode radius is r$_c$<<r$_a$.
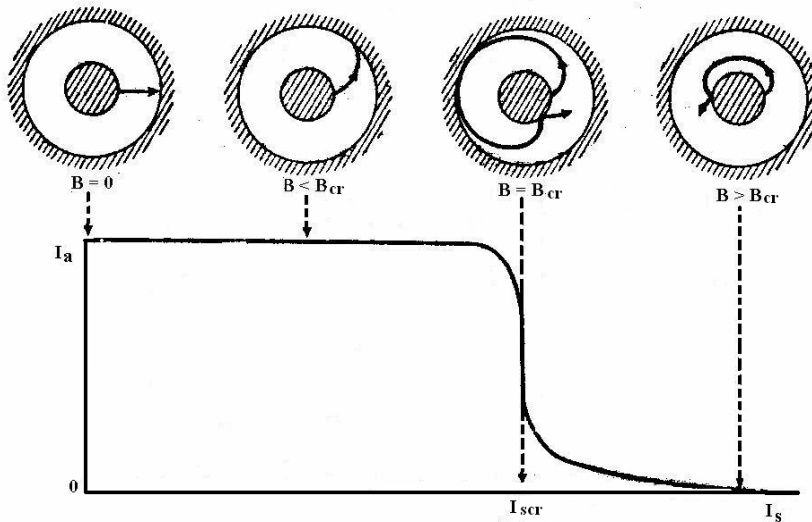
Figure 15. Graphic of the dependence $I_a(I_s)$

If all electrons emitted from the hot cathode have the same velocities, then the critical induction value, $\mathbf{B_{cr}}$ will be the same for all of them and the anode current, $\mathbf{I_a}$ will abruptly become zero (at once) when critical magnetic field is applied. However, the electrons are Maxwell distributed with velocities in different directions when being emitted from the heated cathode. Thus, B_{cr} is not the same for all electrons and the critical field is reached for the different electrons at different values of the solenoid current I_s. As a result, the anode current will not become abruptly zero, but will gradually decrease. Actually, the anode current will never become equal to 0, because the upper part of the diode is closed and few electrons, having velocities parallel to the magnetic induction, will always reach the anode. Further, the anode and the cathode can never be perfectly coaxial and thus, the induction vector will always be not perfectly parallel to the cathode.

EXPERIMENTAL SECTION
I. Schematic view of the measurement set-up.

$\mathcal{E}_1$ and $\mathcal{E}_2$ – sources of a dc-current
A_a – ammeter for the estimation of the anode current $\mathbf{I_a}$
A_s – ammeter for the measurement of the coil current $\mathbf{I_s}$
V – voltmeter
K – cathode
A – anode
R_1 and R_2 – rheostats
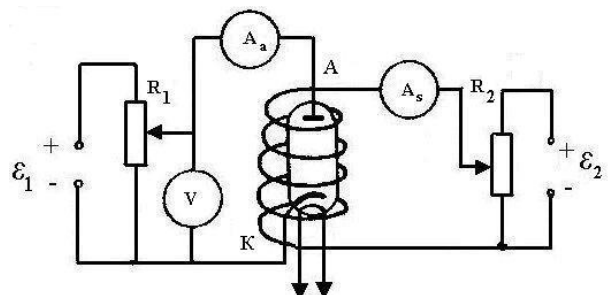
Figure 16. Experimental set-up

II. Measurement of the specific electron charge e/m by using magnetron.
1. The constants of the two ammeters and the voltmeter must be written.

$$K_V = \underline{\qquad}$$

$$K_{Aa} = \underline{\qquad}$$

$$K_{As} = \underline{\qquad}$$

2. The cathode heating is switched on and after the cathode is heated up, different values of the voltage anode-cathode are applied (the voltage U should not exceed 90V to avoid damaging of the diode).

3. The current through the solenoid I_s is changed with an increment of 5 scale divisions and the corresponding values of the anode current I_a are written.

4. All measurements are written down in the table.

№	I_s,		U= .....V I_a,		U= .....V I_a,	
	Scale divisions	A	Scale divisions	A	Scale divisions	A
1.						
2						
3						
4						
5						
6						
7						
8						
9						
10						

5. The graphic Ia vs. Is, $I_a = f(I_s)$ is drawn.
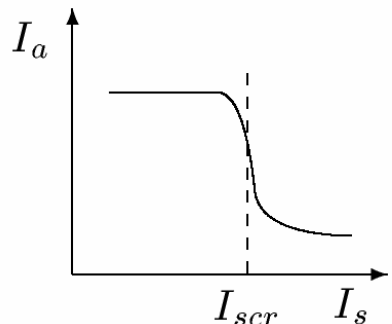
Figure 17. Estimation of I_{scr}

6. From the middle point of the abrupt part of the obtained curve the critical solenoid current $\mathbf{I_{scr}}$ is determined and the critical field $\mathbf{B_{cr}}$ is calculated.

7. The specific charge $\mathbf{e/m}$ is calculated from the expression:

$$\frac{e}{m} = \frac{8U}{\mu_0^2 n^2 I_{скр}^2 r_a^2},$$

$$\frac{e}{m} =$$

where $\mathbf{U}$ is the applied between the anode and cathode voltage;

$\mathbf{n}$ – number of solenoid turns per meter

n=

$\mathbf{r_a}$ – anode radius (the measurement accuracy is much higher than that for the estimation of the current and the voltage and the instrument error for r_a is further neglected)

$r_a=$

8. The relative error for the specific charge is estimated by:

$$\varepsilon = \frac{\Delta\left(\dfrac{e}{m}\right)}{\dfrac{e}{m}} = \pm\left(\frac{\Delta U}{U} + 2\frac{\Delta I_{scr}}{I_{scr}}\right), \qquad\qquad \varepsilon = \frac{\Delta\left(\dfrac{e}{m}\right)}{\dfrac{e}{m}} =$$

where $\mathbf{\Delta U}$ и $\mathbf{\Delta I_{scr}}$ are the instrument errors of the voltmere and ammeter.

$$\Delta U = \qquad\qquad , \qquad \Delta I_{scr} =$$

10. The total error of measurement is obtained from the relation:

$$\Delta\left(\frac{e}{m}\right) = \varepsilon \cdot \frac{e}{m}, \qquad\qquad \Delta\left(\frac{e}{m}\right) =$$

11. The end result is:

$$\frac{e}{m} = \frac{e}{m} \pm \Delta\left(\frac{e}{m}\right), \qquad\qquad \frac{e}{m} =$$

REFRACTION OF LIGHT. REFRACTIVE INDEX. DISPERSION. ABBE'S REFRACTOMETER

THEORETICAL SECTION

When light waves travel through a transparent medium (optical medium), its electric part interacts with the electrons of that medium, causing them to vibrate. The electrons of the medium thus become radiating light waves as the secondary sources. However, the speed of new waves, v, changes accordingly to the optical properties of the particular medium. It is always smaller than the speed of light in vacuum, v<c. All materials are characterized by their ability to slow down the light waves, known as optical refractive index n:

$$n = \frac{v}{c}$$

The refractive index is a dimensionless parameter, equal to 1 for a vacuum and larger than 1 for any other material (e.g. n=1.33 for water). The speed of light in air is only slightly less than c, resulting into the refractive index of 1.0003.

The difference between a light speed in different media results into the change of direction along which the light propagates, refraction. Refraction occurs when the light passes from one medium to a medium with a different index of refraction, except the light that approaches the boundary between the two media perpendicularly. Depending on the properties of an optical medium, some portion of light, approaching the interface at an incident angle α, isreflected back to the first medium while
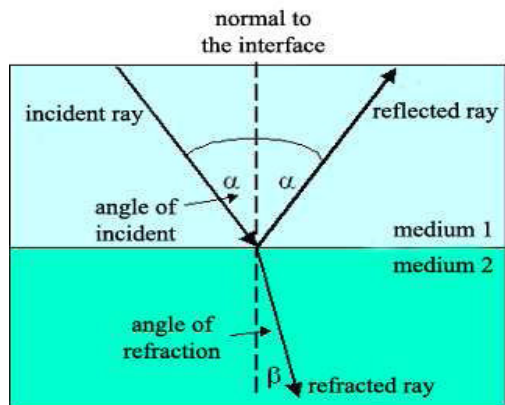
Figure 18. Snell's law

the rest propagates into the other medium at an angle of refraction β. The angles of incidence, reflection and refraction are defined as angles between the particular ray and the interface normal. The refractive angle is determined by the Snell's law:

$$n_1 \sin\alpha = n_2 \sin\beta ,$$

where n_1 is the refractive index of medium 1 and n_2 is the refractive index of medium 2. It is possible to define an optical density for the media of different refractive indices. Medium A has a higher optical density than medium B, if its refractive index is higher than that of medium B. According to the Snell's law, the light ray is "bending towards the normal" (β<α), if it enters the medium

with a higher optical density (Figure 18). When it enters the medium with a lower optical density, it is "bending away from the normal" ($\beta > \alpha$).

Refractive indices can be measured by refractometers. We will use the double prism system called the Abbe's refractometer. It consists of the two optical prisms (illuminating and refracting) with the thin layer of a liquid sample between them. The measuring prism is made of a glass with a high refractive index ($n_2 > 1,75$), which allows this refractometer to measure refractive indices up to $n_1 < 1.75$. The light enters the refractometer from the bottom side of the illuminating prism at many different angles. The upper part of this prism is rough, i.e. it consists of many small areas oriented in different directions. As such, this surface can be imagined as a source shining the light into all directions. Part of this light passes through the sample into the refracting prism, where the biggest possible angle of incident, α is nearly 90°.
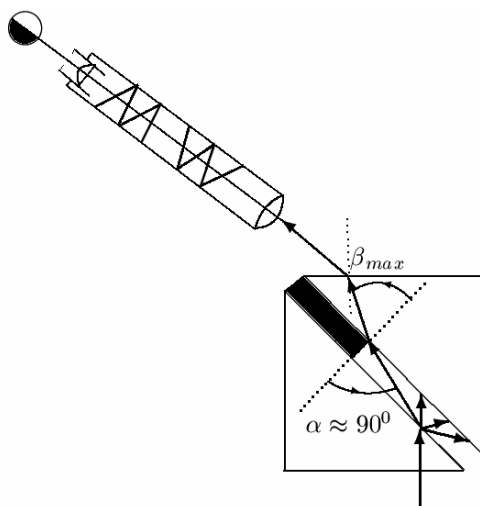
Figure 19. The schematic of Abbe's refractometer

According to the Snell's law, the refraction of this ray is then described by the maximum angle of refraction β_{max}. All rays refracted by the second prism are collected by the objective lens and are looked trough an ocular-lens. The field of vision is a circle, divided to a dark and a bright region. The position of the border between them corresponds to the maximum angle of refraction from Snell's law:

$$\frac{\sin \beta_{max}}{\sin 90^o} = \frac{n_{liquid}}{n_{prism}}$$

and depends on the refractive index of the liquid between the two prisms. The angle of the body tube can be adjusted to match a cross with the border. In this

case the angle of the body tube corresponds to β_{max} and the refractive index can be read from the scale of refractometer.

The refraction index depends on the wavelength of light, because the speed of light waves depends on their wavelength. Light of different colours (different wavelengths) is bending at different angles even if it comes at the same angle of incidence (dispersion). As a result, the white light, that comprises all the wavelengths, produces a rainbow after passing through the optical prism (or droplets of moisture in the atmosphere). However, despite the beauty of a rainbow, this is an unwanted effect in refractive index determination. It causes the smearing of an interface between the illuminated and dark regions in the Abbe's refractometer. To increase the precision of a measurement, it is therefore preferable to use monochromatic light (light of a single wavelength). The most commonly used source is light from a sodium lamp with wavelength equal to 589 nm. The refractive index depends also on the density of the measured sample, which is affected by its temperature. Typically, the refractive index decreases with the decreasing density (increasing temperature). The measurement of a refractive index is therefore reported together with the temperature and the wavelength of light used. The symbol n_D then represents the refractive index measured at t=20 °C using the sodium line D light (low pressure sodium lamp).

The refractive index is an important physical parameter, which is widely used in chemistry. It is commonly used to identify the liquid, or its purity. Often, it is used to determine the concentration of solutions.

EXPERIMENTAL SECTION
I. The determination of the refractive index of glycerine-water solutions

The task consists of the determination of the refractive index of distilled water and different solutions of glycerine and water. The measurement results into calibration curve (refractive index as a function of C% glycerine). This curve will be then used to determine the composition of an unknown mixture of water and glycerine.

Equipment: Abbe's refractometer, pipette, thermometer, different mixtures of glycerine and distilled water at known compositions, mixture of glycerine and distilled water at an unknown composition, filter paper.

II. Experimental procedure:
1. Turn on the light source.
2. Open the double-prism of the refractometer, clean both glass surfaces with a filter paper, and close the double-prism. Use a pipette to fill the space between the two prisms with the distilled water. Turn the refractometer scale knob to get a clear interface between the illuminated and dark regions. Use the

micrometric screw for the additional refinement of the scale, until the clear interface appears. Read out the refractive index from the scale.

3. Open the double-prism and dry out glass surfaces using the filter paper. Repeat the measurement for all glycerine/water mixtures of known and unknown compositions. Fill in the measurement data in the table below:

№	C, %	n
1.		
2		
3		
4		
5		
6		

4. Prepare a calibration curve (i.e. dependence n=f(C% glycerine)). Fit the experimental points with a straight line.

5. Determine the composition of the unknown solution using its refractive index and the calibration curve.

UNIFORMLY ACCELERATED MOTION – COMPUTER ASSISTED MEASUREMENT

APPARATUS

The equipment consists of an Atwood machine connected to a computer and set of weights. The aim of the experiment is to measure the acceleration of masses and compared the results to the prediction of Newton's law.

ATWOOD MACHINE

The Atwood machine is a device which was invented by George Atwood to verify the mechanical laws of motion. The setup is shown in Figure 20.

Two holders are suspended from both ends of a string wrapped around the pulley. They can be loaded by different weights. A large disk is fixed to the pulley. The gears of the disk cross the infrared light of two photogates (as

Figure 20. Setup for measuring acceleration (Atwood machine).

photogates opto-isolators are used). Their signals are shifted in time, which helps to find the direction of rotation.

The right load can be lifted up by a dc motor. The load is fixed in an upper position by an electro-magnet. The motor and the magnet are switched on/off by a computer. The signals from the photogates have a square wave forms, they are send to the Line input of the computer sound card. The front of each pulse is used to trigger the signal which counts the turn of one gear (the triggers are shown as circles in Figure 22). A program written in Delphi is used to read the signals and to show them on the computer screen (see Figure 22 – the signals are in red and blue). The parallel port of the computer is used to control the Atwood machine. A signal from a switch is obtained when the load is in upper position.

EQUATIONS OF MOTION FOR THE ATWOOD MACHINE

In order to calculate the acceleration we must take into account the friction of the pulley and the moment of inertia I of all rotating parts. We will suppose an inextensible string which leads equality of the acceleration of the two weights. We suppose the friction force is proportional to the force on the pulley axis and denote the friction coefficient by k.

The equations of motion for the two loads are:

$$-m_1 a = m_1 g - T_1$$
$$m_2 a = m_2 g - T_2 \qquad (1)$$

The forces acting on each body are shown in Figure 21. The equation of rotation is :

$$I\alpha = T_2 r - T_1 r - F_{тр}\rho$$

The linear and the angular velocities are connected by $v = \alpha r$, and :

$$\mu a = T_2 - T_1 - f_{тр},$$

where $\mu = I/r^2$ is called reduced mass. The force of friction is proportional to the load on the pulley's axis $T_1 + T_2$:

$$\mu a = T_2 - T_1 - k(T_1 + T_2) \qquad (2)$$

Solving these equations one obtains for the acceleration :

$$a = g \frac{m_2 - m_1 - k(m_2 + m_1)}{m_2 + m_1 - k(m_2 - m_1) + \mu} \qquad (3)$$
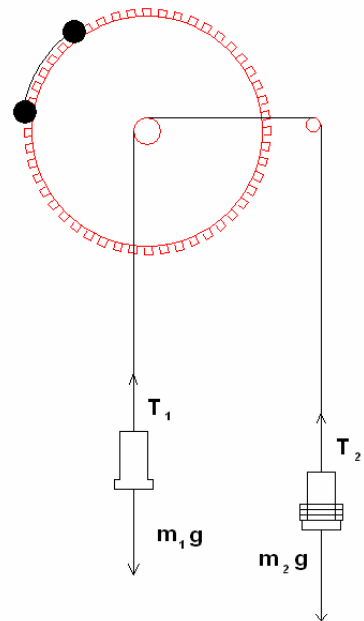
Figure 21. Forces on the loads.

Let's denote $\Delta m = m_2 - m_1$, $2m + \Delta m = m_1 + m_2$, then the acceleration is :

$$a = g\,\frac{\Delta m(1-k) - 2km}{2m + \Delta m(1-k) + \mu} \tag{4}$$

Let expand (4) to first order over Δm :

$$a = \Delta m\,\frac{g}{2m + \mu} - \frac{2gkm}{2m + \mu}, \tag{5}$$

i.e. for small Δm and small friction k we have a linear dependence on the mass difference $a = k_1 \Delta m + k_2$, and measuring the two coefficients k_1 and k_2 we can find the friction coefficient and reduced mass :

$$\mu = \frac{g}{k_1} - 2m \ , \quad k = -\frac{k_2}{2mk_1}$$

EXPERIMENTAL PROCEDURE

1. Put a load on the **right** holder.

2. Start the program from ⊙ icon and **after that** switch on the Atwood machine power supply.

3. Press the program's button with red caption Start. Wait until the load sticks at the upper position and the motor stops. The button caption becomes **Start**. If the load does not stick press the same button until its caption becomes 'Start' and try again.

4. The collected data can be transferred into Excel table choosing the button **Excel**. If you want to keep the measurements, save the data as Excel file before exiting the program. With the two buttons **Stop** and **Clear** you can stop the data acquisition and clear the graph.
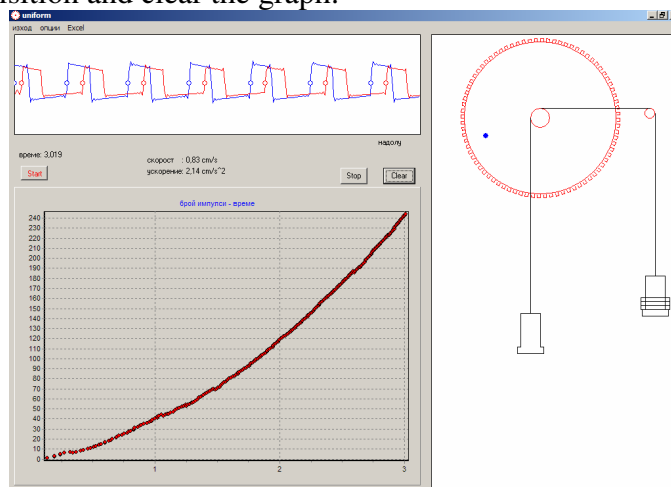
Figure 22. Program screen.

ASSIGNMENTS

1. Find the acceleration at different loads. One count on the graph (one tick) is equal to a displacement of the load by 0.51 mm.

2. Draw how the velocity depends on time.

3. Draw the dependence of the acceleration on the load. Find the parameters μ and k. Compare obtained curve with predicted from (4) and (5).

4. Calculate the maximum speed at different loads using collected data.

5. Calculate the strain of the wire using the maximum speed. Calculate the wire stress, supposing the Young's modulus is 10^{10} Pa.

6. Calculate the reduced mass. Use the data from the table about the rotating parts of the Atwood machine.

Pulley	Disk	Gears	Motor guide circle
density 2.7 g/cm^3 radius 1.5 cm length 3.5 cm $I = \frac{1}{2}mr^2$	density 2.7 g/cm^3 radius 11.15 cm thickness 0.185 cm $I = \frac{1}{2}mr^2$	density 2.7 g/cm^3 radius 11.45 cm thickness 0.185 cm $I = mr^2$	density 0.8 g/cm^3 radius 9.45 cm thickness 0.8 cm width 0.65 cm $I = mr^2$

DENSITY MEASUREMENTS BY PYCNOMETER

THEORETICAL SECTION
Density

The density ρ, is elementary physical property of matter. For a homogeneous object it is defined as the ratio of its mass m to its volume V:

$$\rho = \frac{m}{V} \qquad [1]$$

Numerically it represents the mass per unit volume of matter. As it follows from the definition, the SI unit of density is kg/m^3. However, g/cm^3 is another unit commonly used in the laboratories. The volume of an object increases with increasing temperature, because of the matter's volumetric thermal expansion. Therefore, according to equation [1], the density of an object depends on its temperature, with higher temperature resulting in lower density. Exception is water in temperature range 0-4 °C, for which the density increases with increasing temperature. The density of a gas further depends on the pressure as well. Nevertheless, this effect is negligible in a case of liquid and/or solid matter. There are several experimental methods used for density determination of liquids. We will determine densities by using pycnometer in this assignment.

Density measurements of liquids by pycnometer

Density determination by pycnometer is a very precise method. It uses a working liquid with well-known density, such as water. We will use distilled water, for which temperature dependent values of density ρ_{H2O} are shown in Table 5, Appendix. The pycnometer (see experimental set-up) is a glass flask with a close-fitting ground glass stopper with a capillary hole through it. This fine hole releases a spare liquid after closing a top-filled pycnometer and allows for obtaining a given volume of measured and/or working liquid with a high accuracy.

First we fill the pycnometer with distilled water. According to equation [1], the volume of water that is filling the pycnometer and the stopper is:

Figure 23. Pycnometer

$$V = \frac{m_{H2O}}{\rho_{H2O}} \qquad [2]$$

where m_{H2O} is experimentally determined weight of water (empty pycnometer weight subtracted).

We repeat the procedure for the liquid with unknown density ρ_L and determine its weight m_L (measured weight minus weight of empty pycnometer). The volume V obtained in this measurement is the same as the volume of water determined from equation [2]. It follows alternated equation

$$V = \frac{m_L}{\rho_L} \qquad [3]$$

Combining equations [2] and [3]

$$\frac{m_{H2O}}{\rho_{H2O}} = \frac{m_L}{\rho_L} \qquad [4]$$

yields a relation that provides the density of measured liquid

$$\rho_L = \frac{\rho_{H2O}}{m_{H2O}} m_L \qquad [5]$$
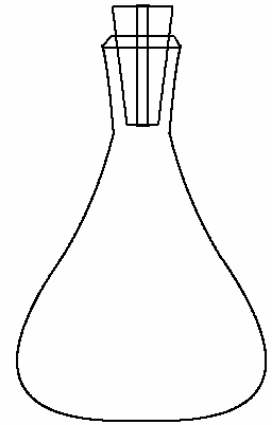
Density measurements of solid matter by pycnometer

Pycnometer can be also used to determine the density of homogeneous solid object that does not dissolve in working liquid (water). First, we need to measure the weight of pycnometer together with inserted object m_0+m_S. We add water and determine the weight m'_{H2O} (measured weight minus m_0+m_S). The volume of added water V'_{H2O} can be obtained as

$$V'_{H2O} = \frac{m'_{H2O}}{\rho_{H2O}} \qquad\qquad [6]$$

The volume of measured solid object V_S is the difference between the volume of water that fills the empty pycnometer V and volume V'_{H2O}

$$Vs = V - V'_{H2O} = \frac{m_{H2O} - m'_{H2O}}{\rho_{H2O}} \qquad\qquad [7]$$

Density of measured object ρ_S can be then calculated as

$$\rho_S = \frac{m_s}{V_s} = \frac{m_s}{m_3 - m_2 + m_1}\rho_{H2O} \qquad\qquad [8]$$

where m_3 – weight of the pycnometer filled only with water

EXPERIMENTAL SECTION

Accuracy of herein described method for density determination of liquid and/or solid matter relies on precise measurements of weight and volume. Since it is important to determine weight of empty pycnometer in its dry state, we do so at the beginning.

1. Determine the weight of the solid objects m_s.

2. Fill in the pycnometer with some objects made of examined material (glass beads or small metal pieces as directed) and measure the weight m_1.

3. Add water in such a way that both the pycnometer and the capillary hole in the stopper are filled with water. Dry the spare water that leaks through the capillary hole with a filter paper and measure total weight m_2.

4. Empty pycnometer and fill it with distilled water only. Use the filter paper to dry the spare water again and measure the weight m_3.

5. Empty pycnometer. Rinse it once with a liquid whose density you are going to determine next. Fill pycnometer with the liquid as previously and measure the weight m_4.

6. Repeat point 5 for several different liquid materials.

7. Clean pycnometer carefully after finishing the experiment. Rinse it with distilled water and let it dry.

8. Measure the laboratory temperature t, which determines the temperature of examined liquids and solid objects.

9. Calculate the weight of water $m_{H2O}=m_3-m_0$, weight of measured liquid $m_L=m_4-m_0$ and determine its density according to equation [5]. Repeat this calculation for all of the measured liquids.

10. In next, calculate the weight of solid object $m_S=m_1-m_0$ and weight of "added" water

$$m'_{H2O} = m_2 - m_1$$

11. Calculate object's volume V_S following the equation [7] and its density ρ_S according to equation [8].

12. The measurement relative error is obtained from

$$\varepsilon = \frac{\Delta\rho_s}{\rho_s} = \frac{\Delta m}{m_s} + \frac{3\Delta m}{m_3 - m_2 + m_1}$$

13. The total error for the density of the solid is determined from:

$$\Delta\rho_s = \varepsilon \bullet \rho_s$$

14. End result:

$$\rho_s \pm \Delta\rho_s =$$

Table 1. The weight of empty pycnometer m_0, pycnometer with solid objects m_1 and pycnometer with solid objects and added water m_2.

m_0 [g]	
m_1 [g]	
m_2 [g]	

Table 2. The weight of pycnometer filled with water, m_3 and that of water only, m_{H2O}

m_3 [g]	m_{H2O} [g]

APPENDICES

1. Physical constants

Speed of light in vacuum	$c=3.10^8$ m/s
Acceleration of gravity	$g=9,80665$ m/s^2
Avogadro number	$N_A=6,022.10^{23}$ mol^{-1}
Gas constant	$R=8,314$ J/(mol.K)
Absolute zero	$0K = -273°C$
Faradey constant	$F=N_A e=9,648.10^4$ C/mol
Electron charge	$e=1,6.10^{-19}$ C
Boltzmann constant	$k=1,38.10^{-23}$ J/K
Standard pressure	$p_0=101325$ Pa
Electron mass	$m_e=9,1.10^{-31}$ kg
Proton mass	$m_p=1,672.10^{-27}$ kg
Neutron mass	$m_n=1,675.10^{-27}$ kg
Electron specific charge	$e/m_e=1,758.10^{11}$ C/kg
Electric constant	$\varepsilon_0=8,85.10^{-12}$ F/m
Magnetic constant	$\mu_0=4\pi.10^{-7}$ H/m
Plank constant	$h=6,6262.10^{-34}$ J.s
Rydberg constant	$R=1,097.10^7$ m^{-1}
Stefan-Boltzmann constant	$\sigma=5,6703.10^{-8}$ W/(m^{2}K^4)
Wien constant	$b=2,8978.10^{-3}$ m.K
Bohr magneton	$\mu_B=9,2741.10^{-24}$ A.m^2
Nuclear magneton	$\mu_N=5,0508.10^{-27}$ Am^2
Gravitational constant	$\gamma=6,672.10^{-11}$ N.m^2/kg^2

2. Curie-points of some ferroelectric and ferromagnetic materials.

Material	Curie-point, °C
Ferroelectric materials	
Barium Titanate $BaTiO_3$	120
Rochelle salt	high: 24; low: - 18
Triglycine sulfate	49
Lead-Zirconium-Titanate (PZT)	230-500
$Bi_4Ti_3O_{12}$	675
Ferromagnetic materials	
Iron	770

Cobalt	1130
Nickel	358
Permalloy 22%Fe, 78%Ni	550
Magnetite Fe_3O_4	570

3. Densities of some solids

Substance	$\rho, 10^3 \frac{kg}{m^3}$	Substance	$\rho, 10^3 \frac{kg}{m^3}$	Substance	$\rho, 10^3 \frac{kg}{m^3}$
Gypsum	2.25-2.87	Iron	7.85	Cork	0.24
Diamond	3.4-3.6	Gold	19.3	Sugar	1.59
Aluminium Bronze	7.7	Potassium	0.86	Lead	11.23-11.44
Aluminium Bronze - rolled	2.62-2.8	Quartz	2.65	Sulfur - rhombic	2.07
Aluminium Bronze – chemically pure	2.58	Constantan	8.8	Sulfur - monoclinic	1.96
Asbestos	1.2-2.8	Ebonite	1.8	Silver	10.41-10.57
Babbitt /Zn-Sb-Cu-Alloy/	7.1	Brass	8.3-8.7	Mica	2.6-3.2
Boron	2.4	Ice	0.83-0.92	Salt	2.8-2.2
Bromine	3.12	Magnesium	1.74	Rock salt	2.28-2.41
Bronze	8.7	Copper	8.6-8.96	Steel	7.7-8.0
Beech - dry	0.62-0.83	Marble	2.52-2.84	Crown glass	2.4-2.6
Paper	0.70-1.15	Sodium	0.97	Flint glass	3.0-5.9
Bismuth	9.76-9.93	Salmiac	1.52	Antimony	6.6
Tunsten	1.91	Nickel	8.4-9.2	White phosphorous	1.8
Wax	0.95-0.99	Tin	7.23-7.5	Zinc	6.86-7.24
Gypsum	2.17-2.31	Paraffin	0.87-0.93	Cast iron	6.6-7.8

crystalline					
Gypsum liquid	1.8	Platinum	21.2-21.7		

4. Gas constants

Substance (* data at 293 K)	Molar mass, 10^{-3} kg / mol	ρ, kg / m^3	Cp / Cv	η, 10^{-5} Pa.s		K, $10^{-2} \dfrac{W}{K.m}$	
				0 °C	20 °C	0 °C	20 °C
Nitrogen	28.016	1.251	1.74	1.67	1.74	2.43	3.15
Ammonia vapour	-	0.771	1.34	0.93	0.97	2.18	2.97
Water vapour	18.016	0.786	1.324	1.28	1.28	2.35	2.38
Hydrogen	2.016	0.0899	1.41	0.84	0.88	16.84	21.60
Carbon monoxide	28.01	1.25	1.40	1.67	1.77	2.16	2.75
Carbon dioxide	44.01	1.977	1.30	1.40	1.45	1.37	1.62
Air – dry	28.96	1.293	1.40	1.72	1.81	2.41	2.57
Oxygen	32.00	1.429	1.385	1.92	2.00	2.44	2.62
Chlorine	70.914	3.22	1.36	1.29	1.32	0.72	-
Helium	4.002	0.1785	1.66	1.89	1.94	14.51	16.70

5. Water density at different temperatures.

t, °C	ρ, kg / m^3	t, °C	ρ, kg / m^3	t, °C	ρ, kg / m^3
0	999.87	12	999.52	24	997.32
1	999.93	13	999.40	25	997.07
2	999.95	14	999.27	26	996.81
3	999.99	15	999.13	27	996.54
4	1000	16	998.97	28	996.26

5	999.99	17	998.80	29	995.97
6	999.97	18	998.62	30	995.67
7	999.93	19	998.43	31	995.37
8	999.88	20	998.23	32	995.05
9	999.81	21	998.02	33	994.72
10	999.73	22	997.80	34	994.40
11	999.63	23	997.57	35	994.06

6. Resistivity and thermal coefficient of resistance of some metals.

Metals and alloys	ρ $10^{-6}\,\Omega.m$	α $10^{-3}\,K^{-1}$	Metals and alloys	ρ $10^{-6}\,\Omega.m$	α $10^{-3}\,K^{-1}$
Silver	0.015	4.1	Cobalt	0.056	6.5
Copper	0.0155	4.3	Cadmium	0.073	4.2
Gold	0.0204	4.0	Iron	0.10	5.6
Aluminium	0.024	4.7	Osmium	0.096	4.0
Magnesium I	0.046	-	Palladium	0.109	3.6
Magnesium II	0.038	4.1	Platinum	0.098	6.92
Radium	0.037	4.4	Tin	0.100	4.6
Iridium	0.0458	4.1	Tantalum	0.120	3.5
Tungsten	0.0491	4.8	Bismuth	0.120	4.4
Molybdenum	0.054	4.7	Lead	0.188	4.0
Zinc	0.048	4.2	Mercury	0.958	0.99
Nickel	0.0605	6.7	Hard steel	0.45	1.5
Brass	0.08	0.05	Iron-Alloys	0.50	0.9

7. Deformation moduli

Material	Young's Modulus E, GPa	Shear Modulus G, GPa
Aluminium	70	26
Tungsten	380	140
Iron (Steel)	200	81
Copper	130	40
Lead	16	6
Silver	74	27
Glass	60	30

8. Sound velocity in solids at 20 °C (in m/s).

Material	In bar	Longitudinal Wave	Transversal Wave
Aluminium	5080	6260	3080
Iron	5170	5850	3230
Gold	2030	3240	1200
Copper	3710	4700	2260
Brass	3490	4430	2123
Nickel	4785	5630	2960
Lead	1200	2160	700
Silver	2640	3600	1590
Steel	5050	6100	-
Zinc	3810	4170	2410

9. Mean radius of atoms and molecules.

Name (Symbol)	Mean Radius, 10^{-10}m
Nitrogen (N$_2$)	3.7
Argon (Ar)	3.5
Hydrogen (H$_2$)	2.7
Oxygen (O$_2$)	3.5
Helium (He)	2.0

10. Specific heat capacity.

Gas	Cp/Cv	c_V, $\dfrac{kJ}{kg.K}$
Nitrogen (N$_2$)	1.40	0.74
Argon (Ar)	1.67	0.32
Water vapour (H$_2$O)	1.82	1.38
Hydrogen (H$_2$)	1.41	10.1
Air	1.40	0.73
Carbon dioxide (CO$_2$)	1.30	0.65
Oxygen(O$_2$)	1.40	0.65
Helium (He)	1.63	3.14
Liquids	c_V, $\dfrac{kJ}{kg.K}$	
Benzol	1.72	
Water	4.19	
Glycerine	2.43	
Mercury	0.138	

Ethanol	2.51
Solids	c_V, $\dfrac{J}{kg.K}$
Aluminium	896
Tin	230
Ice	2100
Copper	386
Iron	500
Brass	386
Lead	126
Silver	234
Steel	460
Zinc	391

11. Relative dielectric constants.

Dielectric	Relative dielectric constant ε_r
Water	81
Air	1.0058
Natural gas	2.0
Ebonite	2.7
Paraffin	2.0
Plexiglass	3.5
Polyethylene	2.3
Porcelain	6.0
Mica	7.5
Ethanol	26
Glass	6.0

12. Resistivity ρ and thermal coefficient of the resistance α.

Conductor	$\rho(at\ 20°C)$ $10^{-9}\,\Omega.m$	α $10^{-3}\,\mathrm{K}^{-1}$
Aluminium	25	4.5
Tungsten	50	4.8
Graphite	$3.9.10^{3}$	-0.8
Iron	90	6.5
Constantan	$4.8.10^{2}$	0.02
Copper	17	4.3
Nichrome	10^{3}	0.26
Silver	15	4.1

13. Relative magnetic permeability of para- und diamagnetic materials.

Paramagnetic	$\mu-1,$ 10^{-6}	Diamagnetic	$\mu-1,$ 10^{-6}
Nitrogen	0.013	Hydrogen	-0.063
Air	0.38	Benzol	-7.5
Oxygen	1.9	Water	-9.0
Aluminium	23	Copper	-10.3
Tungsten	176	Rock Salt	-12.6
Platinum	360	Quartz	-15.1
Liquid Oxygen	3400	Bismuth	-176

14.   Refraction index n (mean values for the visible range of the electromagnetic spectrum).

Material/Substance	Index of Refraction n
Water	1.33
Air	1.00029
Diamond	2.42
Ice	1.31
Carbon Disulfide	1.63
Glass	1.5-1.9

15. Ionisation potential φ_i of atoms and molecules.

Atom	φ_i, V
Hydrogen, H	13.5
Helium, He	24.5
Lithium, Li	5.4
Beryllium, Be	9.3
Boron, B	8.3
Carbon, C	11.3
Nitrogen, N	14.5
Oxygen, O	13.6
Fluorine, F	17.4
Neon, Ne	21.5
Sodium, Na	5.1
Mercury, Hg	10.4

16. Metals – characteristic quantities.

Metal	Work function A, eV	Fermi-Energy W_F, eV	Crystalline lattice		Debye-temperature θ_D, K	Melting point t_m, °C
			Elementary cell	Lattice constant a, Å		
Aluminium	4.25	11.8	Cubic face-centered	4.04	374	658.7
Barium	2.49	-	Cubic body-centered	5.02	116	704
Beryllium	3.92	-	-	-	1000	1278
Lead	4.0	-	Cubic face-centered	4.94	89	327.5
Cesium	1.81	1.53	Cubic body-centered	6.10	60	28.5
Iron	4.31	-	Cubic body-centered	2.86	467	1535
Gold	4.30	5.54	Cubic face-centered	4.07	180	1063
Potassium	2.22	2.14	Cubic body-centered	5.31	132	62.3
Cobalt	4.41	-	-	-	-	1480
Copper	4.40	7.04	Cubic face-centered	3.61	320	1033
Lithium	2.38	4.72	Cubic face-centered	3.50	404	186
Magnesium	3.64	-	-	-	-	650
Molybdenum	1.3	-	Cubic body-centered	3.14	357	2620
Sodium	2.35	3.12	Cubic body-centered	4.28	226	97.5
Nickel	4.50	-	Cubic face-centered	3.52	425	1452
Platinum	5.32	-	Cubic face-centered	3.92	212	1775
Silver	4.3	5.51	Cubic face-centered	4.08	210	960
Titanium	3.95	-	-	-	-	1720
Bismuth	4.42	-	-	-	-	271

Tungsten	4.54	-	Cubic body-centered	3.16	310	3370
Zinc	4.24	11.0	-	-	250	419.4
Tin	4.38	-	-	-	-	231.9

17. Relative atomic masses of some isotopes.

Atomic number Z	Isotope	Mass	Atomic number Z	Isotope	Mass
0	n	1.00867	6	C^{11}	11.01143
1	H^1	1.00783		C^{12}	12.00000
	H^2	2.01410		C^{13}	13.00335
	H^3	3.01605	7	N^{13}	13.00574
2	He^3	4.01603		N^{14}	14.00307
	He^4	4.00260		N^{15}	15.00011
3	Li^6	6.01513	8	O^{15}	15.00307
	Li^7	7.01601		O^{16}	15.99491
4	Be^7	7.01693		O^{17}	16.99913
	Be^8	8.00531	9	F^{19}	18.99840
	Be^9	9.01219	10	Ne^{20}	19.99244
	Be^{10}	10.01354	11	Na^{23}	22.98977
5	B^{10}	10.01294		Na^{24}	23.99097
	B^{11}	11.0930	12	Mg^{24}	23.98504

18. Half-value period of some isotopes.

Atomic number Z	Isotope	Type of Decay	Half-value Period
12	Magnesium Mg^{21}	β^-	10 min = 600 s
12	Phosphorus P^{32}	β^-	14.3 days = $1.24.10^6$ s
27	Cobalt Co^{60}	β^-, γ	5.3 years = $1.7.10^8$ s
38	Strontium Sr^{90}	β^-	28 years = $8.85.10^8$ s
53	Iodine I^{131}	β^-, γ	8 days = $6.9.10^5$ s
77	Iridium Ir^{142}	β^-, γ	75 days = $6.5.10^6$ s
84	Polonium Po^{210}	α	138 days = $1.98.10^7$ s
86	Radon Rn^{222}	α	3.8 days = $3.28.10^5$ s
88	Radium Ra^{226}	α, γ	1620 years = $5.12.10^{10}$s
89	Actinium Ac^{225}	α	10 days = $8.64.10^5$ s
90	Thorium Th^{229}	α, γ	7.10^3 years = $2.2.10^{11}$ s
92	Uranium U^{238}	α, γ	$4.5.10^9$ years= $1.4.10^{17}$s

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